

Thermal drag in forced duct flows

ZENG-YUAN GUO and WEI-HONG BU

Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China

(Received 15 August 1989 and in final form 18 February 1990)

Abstract—An investigation of the thermal drag phenomena is conducted both analytically and experimentally. The existence of thermal drag gives rise to the decrease of both the mass flow rate and Mach number in the flow systems. Defining the thermal drag coefficient C_t and dimensionless heating number He , an approximate correlation between them is obtained for one-dimensional, inviscid duct flow with simple heating. The dependence of the ratio of friction-caused pressure drop ΔP_f to heating-caused ΔP_h on the heating number is presented for turbulent duct flow, and clarifies the relative importance of the thermal drag to the viscous drag for different heating intensity.

1. INTRODUCTION

HEAT TRANSFER and fluid flow appear simultaneously in many problems of engineering interest, such as heat transfer equipment used in the power, chemistry, and metallurgical industries. Because of the great influence of fluid flow on heat transfer, fluid mechanics is usually regarded as the basis of convective heat transfer in research work, as well as in university teaching. As a result, a large body of research papers has been contributed to it. In the meantime, only few people paid attention to the reverse effect, that is, the effect of a thermal process on fluid flow. If any, this reverse effect was usually summed up as the influence of variation of fluid properties with temperature. In the existing literature there were three schemes for taking account of the variable property effect: (a) reference temperature method or property ratio method, of which the core lies in making a correction to the constant property results (friction factor C_f , Nusselt number Nu , etc.) [1-5]; (b) transformation of variables, such that the effect of fluid property variations in governing equations can be diminished, even eliminated in rare cases [6-7]; (c) numerical approach, the equations for the coupled flow being solved numerically to obtain the flow and temperature field and C_f , Nu as well [8, 9]. The present work devotes itself to investigating the heating effect on the flow properties in duct flow with an emphasis on its physical mechanism and quantitative description as a class effect.

2. THERMAL DRAG

It is a well-known fact that the frictional shearing stress inevitably creates a pressure drop in viscous duct flows. Similarly, the heat addition to the inviscid duct flow may also cause a pressure drop along the flow direction. Abramovich [10] indicated the pressure drop due to heating in the one-dimensional, inviscid duct flow, and referred to the stagnation pressure drop as the thermal drag without further analysis. Shapiro [11] investigated the choking phenomenon in

duct gas flow caused by various factors, including the heating effect. Based on the solution of the governing equation for the one-dimensional duct subsonic flow with simple heating. The physical mechanism of thermal drag and its influence on the behaviour of gas flow in a duct was clarified in refs. [12, 13].

2.1. Physical mechanism

Consider a case, as shown in Fig. 1, where a constant area duct with heat generation in it is fed by an extensive vessel at pressure P_0 , which is greater than the back pressure P_2 . Thus, a continuous gas stream from the vessel through the duct to the ambient occurs. Under the simplifications that (i) there is no heat and work exchange between the system and environment; (ii) viscosity is negligible; (iii) the flow is one-dimensional, the gas flow in the duct can then be described by the following one-dimensional, steady and inviscid conservation equations by means of flow properties at sections 1 and 2:

continuity

$$\rho_1 V_1 = \rho_2 V_2; \quad (1)$$

momentum equations

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 \quad (2)$$

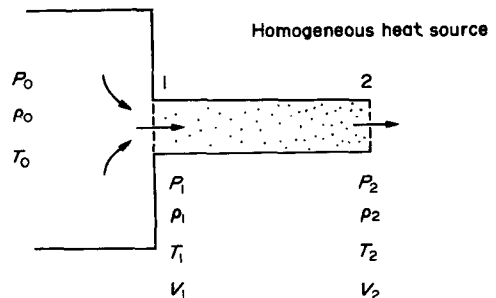


FIG. 1. One-dimensional drag in the constant-area duct.

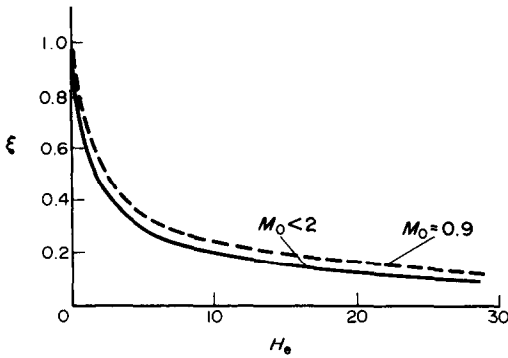


FIG. 2. Flow rate ratio vs heating number.

This expression obviously suggests that heating the flowing gas can reduce the mass flow rate in the duct. Such an effect may also be called thermal clogging. Moreover, it can be found from equation (17) that: (1) $\xi = 1$, as $He = 0$ (no clogging); (2) $\xi = 1/2$, as $He = 1$ (half clogging); (3) $\xi \rightarrow 0$, as $He \rightarrow \infty$ (entire clogging). Consequently, the fact that the heat addition to the flowing gas will give rise to a reduction in mass flow rate, even to the entire clogging in the extreme case, is good evidence for the presence of the thermal drag in duct flow.

The graphic illustration of expression (17) is given in Fig. 2 and the results of the numerical approach to equations (1)–(9) are also plotted in the same figure for comparison. Good agreement between them can be seen apart from the case of $M > 0.2$.

One thing should be noted that thermal clogging is distinct from the choking phenomenon [11] that the adjustment in inlet flow rate is necessary only if the leaving Mach number of the duct flow has reached unity and any further heat is added to the duct flow. However, we have the case that the mass flow rate in the duct will decrease as long as any heat is added to the subsonic gas flow.

2.2.2. *Mach number.* The heating effect on Mach number of gas flow in the duct can also be obtained by means of the numerical solution of the governing equations (1)–(9). Some results about it are plotted in Figs. 3 and 4. It is found that the heat addition to the

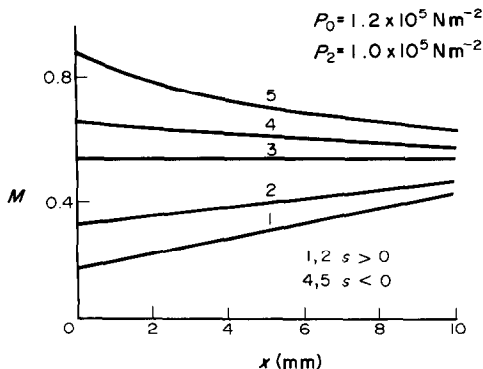


FIG. 3. Mach number variation along the duct.

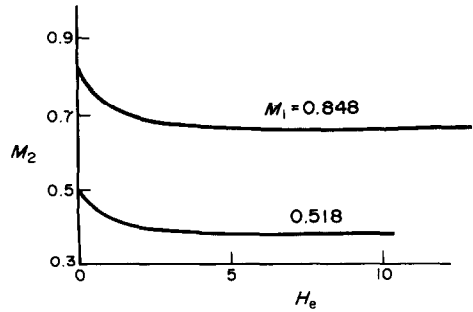


FIG. 4. Leaving Mach number variation with heating intensity.

flowing gas results in an increase of the Mach number along the duct. This remark may be found in most of the books on dynamics of compressible fluid flow. However, Fig. 4 shows that the outlet Mach number, M_2 , decreases with gas heating and gradually approaches to a limiting value less than unity as the heating intensity goes to infinity. At the very beginning, this finding seemingly conflicts with the conventional knowledge that the outlet Mach number always increases with increasing heat addition to the duct flow before it reaches unity (for the subsonic flow). In fact, this difference in the above concluding remarks originates from the different formulations for problems discussed rather than reflects the conflicting physical phenomena. The well-known remark about the heating effect on outlet Mach number is based on the assumption [11, 12] of unchanged flow properties at the duct inlet, which was, perhaps, mainly against the background of problems related to ram-jet engines in the 1950s. The present findings, however, are based on the condition of fixed inlet flow stagnation properties and back pressure conforming to the general engineering problems with a gas reservoir as a gas supply. It is due to the presence of thermal drag that the mass flow rate and consequent outlet Mach number reduce with heating intensity.

2.2.3. *Other properties.* Some results about heating effects on other flow properties (direction of changes in flow properties) are listed in Table 1. For the sake of contrast, heating effects on conventional flow systems are also given in Table 1. It can be seen that here a set of flow patterns, different from the conventional one, of duct flow under heating is provided.

3. THERMAL DRAG COEFFICIENT

3.1. Definition

In view of the fact that the heat addition to the gas flow tends to produce a pressure drop, similar to the friction factor or form drag coefficient used commonly in fluid mechanics, a thermal drag coefficient can be defined as

$$C_t = \frac{P_1 - P_2}{\rho_2 V_2^2 / 2} \quad (18)$$

Table 1. Heating (or cooling) effects on subsonic flow properties

	P_0, T_0, P_2 fixed (present work)		P_1, V_1, T_1 fixed (Shapiro [11])	
	Heating	Cooling	Heating	Cooling
Outlet velocity	increase	decrease	increase	decrease
Outlet Mach number	decrease	increase	increase	decrease
Outlet pressure	unchanged	unchanged	decrease	increase
Inlet velocity	decrease	increase	unchanged	unchanged
Inlet Mach number	decrease	increase	unchanged	unchanged
Inlet pressure	increase	decrease	unchanged	unchanged

where C_t denotes the thermal drag coefficient, $(P_1 - P_2)$ the pressure drop between duct inlet and outlet cross-section, and $V_2^2/2$ the dynamic head of the flowing gas at the duct outlet.

3.2. Analytical expression

The rearrangement of results from the solution of the governing equations yields

$$C_t = \frac{1 + He - \frac{(1 + kM_1^2)\left(1 + \frac{k-1}{2}M_2^2\right)}{(1 + kM_2^2)\left(1 + \frac{k-1}{2}M_1^2\right)}}{1 + He} \quad (19)$$

where the Mach numbers, M_1 and M_2 , are functions of the pressure ratio, P_0/P_2 , and the heating number, He . That means, the thermal drag coefficient, C_t , is a function of the pressure ratio and heating number.

If the pressure ratio is much less than the critical pressure ratio (or the Mach number is much less than unity), expression (19) can be reduced to

$$C_t = \frac{2He}{1 + He} \quad (20)$$

It is found that the thermal drag coefficient, C_t , depends only on the heating number, He , for the duct flow with small Mach number.

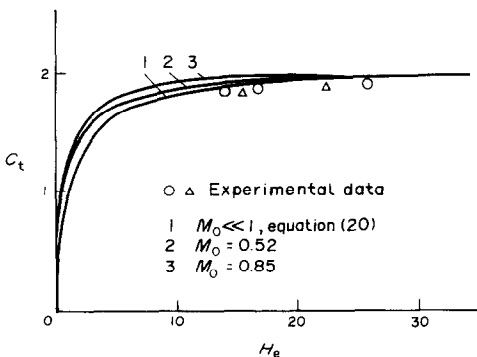


FIG. 5. Dependence of the thermal drag coefficient on the dimensionless heating number.

Figure 5 is the graphic illustration of expressions (19) and (20), where the Mach number of the gas flow without heating, M_0 , acts as a parameter. From Fig. 5, the thermal drag coefficient, C_t , is only a weak function of the Mach number, M_0 . It turns out that the approximate expression (20) can be employed with high validity unless the Mach number M_0 is beyond 0.2. In the case of very large heating number, the thermal drag coefficient, C_t , approaches a limiting value of 2, regardless of expression (19) or (20). It means that the pressure drop due to heating in this case is twice that of the dynamic head at the duct outlet.

It is easy to see that expressions (19) and (20) or the family of curves in Fig. 5, in particular, expression (20) provides a very simple way to predict the pressure drop in the duct flow due merely to heating. The procedure for evaluating the pressure drop of this kind is similar to that for predicting the pressure drop due to friction. Their analogy is given as follows:

$$Re \rightarrow C_f \rightarrow \Delta P_f$$

$$He \rightarrow C_t \rightarrow \Delta P_t$$

where C_f and C_t are the friction drag coefficient and the thermal drag coefficient, and ΔP_f and ΔP_t the pressure drop due to friction and due to heating, respectively. It may be expressed that the importance of expression (20) to predicting the pressure drop of the duct flow is similar to that of the expression $C_f = 16/Re$.

3.3. Relative importance of the thermal drag to the viscous drag

In view of the fact that the viscous drag in the duct flow always exists in practical problems, to clarify the relative importance of the thermal drag to the viscous drag is very crucial for engineering application. Hence, this problem was investigated [11] for the compressible turbulent flow in a circular duct under heating. Its governing equations are

$$\frac{\partial}{\partial x}(r\rho u) + \frac{\partial}{\partial r}(r\rho v) = 0 \quad (21)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\frac{dP}{dx} + \frac{\partial}{\partial r} \left[r(\mu + \rho \epsilon_m) \frac{\partial u}{\partial r} \right] \quad (22)$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial r} = \frac{\partial}{\partial r} \left\{ r \left(\frac{\mu}{Pr} + \rho \frac{\epsilon_m}{Pr_t} \right) \frac{\partial H}{\partial r} \right. \\ \left. + r \left[\mu \left(1 - \frac{1}{Pr} \right) + \rho \epsilon_m \left(1 - \frac{1}{Pr_t} \right) \right] u \frac{\partial u}{\partial r} \right\} + s \quad (23)$$

where u and v are the axial and radial components of gas velocity, H the gas enthalpy, and ϵ_m and Pr_t the turbulent viscosity and turbulent Prandtl number, respectively. s is the heating intensity.

This is a problem of strongly coupled duct flow which continues to develop indefinitely in the flow direction, as the heat is added to and the gas expands continuously along the duct [15]. Consequently, this problem should be taken as a developing case. The GEMIX program [16] with some modification has been used in ref. [14] for the numerical approach of turbulent duct flows under heating. What is different from the conventional numerical results is to separate the pressure drop, ΔP_f , due to friction and the pressure drop, ΔP_v , due to heating from the total pressure drop ΔP in the duct. The pressure drop due to friction, ΔP_f , can be obtained through the integration of the friction shearing stress over the inner surface of the duct, while the subtraction of ΔP_f from the total pressure drop ΔP yields the pressure drop due to heating, ΔP_v .

After the arrangement of numerical results, the dependence of $\Delta P_f/\Delta P_v$, which demonstrates the relative importance of the thermal drag to the viscous drag, on the heating number, He , is small, while ΔP_f can even be neglected, as He exceeds a certain value and L/D is not too large (e.g. $He > 1.5$, $L/D < 10$). In addition, ΔP_f and ΔP_v are of the same order of magnitude in the wide range of He . All these findings not only suggest that the thermal drag is important in some cases, but also indicate how one takes the thermal drag into consideration.

Lu [17] conducted a further study on the problem of combined thermal drag and viscous drag. In terms

of the simplified analysis and numerical results of governing equations for coupled duct flow, it is shown that expression (20)

$$C_t = \frac{2He}{1+He}$$

still approximately holds for the coupled flow (gas flow with viscosity and heating taken into consideration). This remark implies that the impact of heating-induced viscosity variation on the thermal drag coefficient is approximately negligible, although gas heating can effect remarkably the viscous drag coefficient C_f . This finding is of significance for engineering practice, because the simple expression (20) can also be used to predict the drop, ΔP_v , even in the case of the existence of both the viscosity and thermal drag, and furthermore, the total pressure drop ΔP may be obtained without calculating the friction pressure drop ΔP_f by means of the following relation:

$$C_{\text{total}} = \frac{\Delta P_1 + \Delta P_f}{0.5\rho_2 V_2^2} = C_t \left(1 + \frac{\Delta P_f}{\Delta P_1} \right) \\ = \left(1 + \frac{\Delta P_f}{\Delta P_1} \right) \frac{2He}{1+He} \quad (24)$$

where C_{total} represents the total drag coefficient, and the ratio, $\Delta P_f/\Delta P_1$, may be obtained from Fig. 6.

4. EXPERIMENT

The experiment for the purpose of demonstrating the thermal drag and its impact on flow properties was carried out in the test flow system, as shown in Fig. 7. The test sections made of copper are constant area ducts 37 mm in length, 6 and 8 mm i.d. which are connected to a tank. The operating gas (argon) comes from the gas reservoir. The d.c. burning arc inside the test duct acts as the heat supply for gas heating. The heating intensity is controllable by changing the arc current so that the relation between the thermal drag and the heating number can be obtained. The stagnation pressure P_0 in the tank was maintained at the values desired during the experiments. The measured parameters cover the tank P_0 , inlet and outlet static pressure, P_1 and P_2 , duct flow rate, G , arc voltage and current, U and I , the heating flow Q taken away by the cooling water which is used to prevent the copper duct from melting, and so on. The experimental results [12] on the pressure drop ($P_1 - P_2$) between the test duct inlet and outlet and the mass flow rate \dot{m} in the duct with arc current I (it reflects the heating intensity) are replotted in Fig. 8. It can be seen that: (i) the pressure drop ($P_1 - P_2$) caused by gas heating is much greater than the friction induced pressure drop (ΔP_f is 85 mm H₂O at room temperature); (ii) the thermal drag increases and the mass flow rate in the duct decreases obviously with rising heating intensity; (iii) the thermal drag phenomenon is more profound for the duct flow

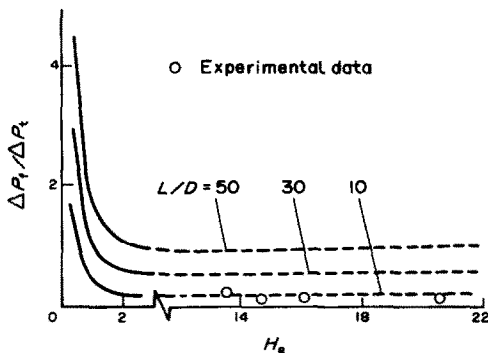


FIG. 6. Pressure drop ratio vs heating number.

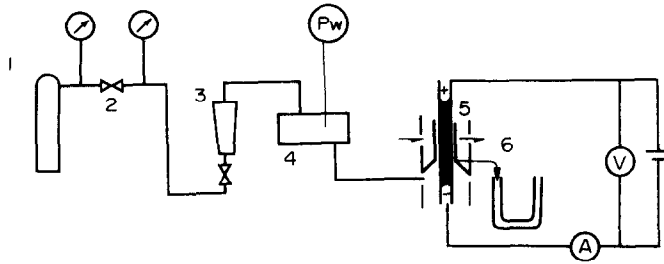


FIG. 7. Test flow system: 1, gas reservoir; 2, throttle valve; 3, flowmeter; 4, tank; 5, electric arc; 6, water.

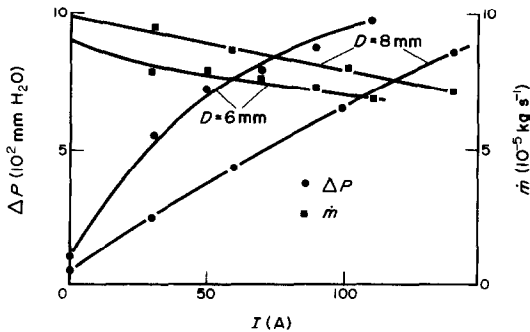


FIG. 8. Variation in the pressure drop and flow rate under arc heating.

with small length-diameter ratio. The results can be rearranged to indicate the correlation between the thermal drag coefficient and the heating number, and the correlation between the pressure drop ratio $\Delta P_t/\Delta P_f$ and the heating number, that are also given in Figs. 5 and 6, respectively, for comparison. The fair agreement between the experimental data and the analytical results shows the applicability of the approximate expression (20) for the duct flow with small Mach number.

5. CONCLUDING REMARKS

(1) The heat addition to the inviscid duct flow will produce a pressure drop along the flow direction, which is referred to as the thermal drag. The existence of the thermal drag leads to the decrease of the mass flow rate and Mach number in the present flow system, which is different from the conventional knowledge.

(2) The thermal drag coefficient C_t is a function of both the heating number and the Mach number (or pressure ratio), but only a function of the heating number in the case of small Mach number. The simple correlation between the thermal drag coefficient and the heating number, equation (20), provides a simple way to predict the pressure drag in the duct flow due merely to the gas heating, similar to the prediction of the friction-induced pressure drop by the simple correlation between the friction drag coefficient and Reynolds number.

(3) Numerical results on the ratio of the pressure drop due to friction to that due to heating, which depends on the heating number and the length-diameter ratio of the duct, clarify the relative importance of the thermal drag, and tell us in what cases we have to take the thermal drag into consideration.

(4) Preliminary experiments show that the pressure drop and flow rate variation in the duct caused by gas heating are evident and the approximate expression (20) is applicable for the prediction of the pressure drop due purely to heating.

(5) The concept of the thermal drag and the simple correlation between the thermal drag coefficient and the heating number, as a class effect, are expected to find their applications in the combustor, convective boiling equipment, arc heater and other kinds of thermal equipment.

Acknowledgement—The author is indebted to Prof. B. X. Wang for stimulating this topic and valuable discussions. The author would like to thank the China Natural Science Foundation for financial support.

REFERENCES

1. E. R. G. Eckert, Engineering relation for friction and heat transfer to surface in high velocity flow, *J. Aero. Sci.* 585–587 (August 1955).
2. K. T. Yang, Laminar forced convection of liquids in tubes with variable viscosity, *J. Heat Transfer* **84**, 352–362 (1962).
3. W. M. Kays and W. B. Nicoll, Laminar flow heat transfer to a gas with large temperature difference, *J. Heat Transfer* **85**, 329–385 (1963).
4. B. S. Peturhov, *Advances in Heat Transfer*, Vol. 6, pp. 503–564. Academic Press, New York (1970).
5. C. A. Sleicher and M. W. Rouse, A convenient correlation for heat transfer to constant and variable property fluids in turbulent pipe flow, *Int. J. Heat Mass Transfer* **18**, 677–683 (1975).
6. C. R. Illingworth, Steady flow in the laminar layer of a gas, *Proc. R. Soc. A* **199**, 533–543 (1949).
7. K. Stawartson, Correlated compressible and incompressible boundary layers, *Proc. R. Soc. A* **200**, 84–92 (1949).
8. C. B. Cohen and E. Reshotko, The compressible laminar boundary layer with heat transfer and arbitrary pressure gradient, NACA Rep. No. 1294 (1956).
9. S. V. Patanker, *Numerical Heat Transfer and Fluid Flow*. McGraw-Hill, Washington, DC (1980).

10. G. N. Abramovich, *Applied Gas Dynamics* (in Russian). National Press, Moscow (1953).
11. A. H. Shapiro, *The Dynamics and Thermodynamics of Compressible Fluid Flow*. Ronald Press, New York (1953).
12. Z. Y. Guo and W. H. Bu, Thermal drag in fluid flow system. In *Heat Transfer Science & Technology* (Edited by B. X. Wang). Hemisphere, New York (1987).
13. Z. Y. Guo, Thermal drag and thermal roundabout flow in convective problems, Keynote Paper, *Proc. 8th Int. Heat Transfer Conf.*, Vol. 1, pp. 59–68 (1986).
14. W. H. Bu and Z. Y. Guo, Combined thermal drag and friction drag (in Chinese), *J. Engng Thermophys.* 7(2), 252–254 (1986).
15. T. Cebeci and P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer*. Springer, New York (1984).
16. D. B. Spalding, *Gemix—A Computer Program for Two-dimensional Parabolic Phenomena*. Pergamon Press, London (1977).
17. D. M. Lu, Numerical analysis of the thermal drag in duct flow (in Chinese), Diploma Thesis of Tsinghua University (1986).

TRAINEE THERMIQUE DANS LES ECOULEMENTS FORCES DANS LES TUBES

Résumé—On étudie analytiquement et expérimentalement les phénomènes de trainée thermique. Ceux-ci conduisent à une diminution à la fois du débit-masse et du nombre de Mach. En désignant le coefficient de trainée thermique par C_t et le nombre adimensionnel de chauffage par He , on obtient une formule approchée entre eux pour un écoulement unidirectionnel sans viscosité avec chauffage. On présente, pour un écoulement turbulent dans un tube, la dépendance du rapport de la perte de pression par frottement ΔP_f à celle causée par le chauffage ΔP_c , vis-à-vis du nombre He ; on éclaireit l'importance relative de la trainée thermique par rapport à la trainée visqueuse pour différentes intensités de chauffage.

DER THERMISCH BEDINGTE WIDERSTAND IN ERZWUNGENER KANALSTRÖMUNG

Zusammenfassung—Das Phänomen des thermischen Widerstandes wird analytisch und experimentell untersucht. Aufgrund des thermischen Widerstandes ergibt sich eine Abnahme des Massenstroms und der Mach-Zahl in Strömungssystemen. Mit Hilfe des Koeffizienten für den thermischen Widerstand C_t und der dimensionslosen Heizzahl He ist es möglich, eine Korrelationsgleichung für eine eindimensionale reibungsfreie Kanalströmung mit einfacher Beheizung zu formulieren. Für turbulente Kanalströmung wird der Einfluß des Verhältnisses aus reibungsbedingtem Druckabfall ΔP_f und beheizungsbedingtem Druckabfall ΔP_c auf die Heizzahl vorgestellt. Dieses Ergebnis zeigt die relative Wichtigkeit des thermischen Widerstandes im Vergleich zum reibungsbedingten Widerstand für unterschiedliche Intensität der Beheizung.

ТЕПЛОВОЕ СОПРОТИВЛЕНИЕ ПРИ ВЫНУЖДЕННЫХ ТЕЧЕНИЯХ В КАНАЛАХ

Аннотация—Аналитически и экспериментально исследуются явления теплового сопротивления. Существование теплового сопротивления приводит к уменьшению массового расхода и числа Маха в проточных системах. Получена приближенная зависимость между коэффициентом теплового сопротивления C_t и безразмерным числом нагрева He для одномерного течения без учета вязкости при простом нагреве. Описывается зависимость отношения перепада давления из-за трения ΔP_f к перепаду давления из-за нагрева ΔP_c от числа нагрева в случае турбулентного течения в канале, и объясняется относительная важность отношения теплового и вязкостного сопротивлений при различной интенсивности нагрева.